

Direct Quadratic Minimization using Magnetic Field-based Computing

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Abstract—We explore an unconventional front in computing, which we call magnetic field-based computing (MFC), that harnesses the energy minimization aspects of a collection of nanomagnets to solve directly quadratic energy minimization problems, such as those arising in computationally intensive computer vision tasks. The Hamiltonian of a collection of bipolar nanomagnets is governed by the pairwise dipolar interactions. The ground state of a nanomagnet collection minimizes this Hamiltonian. We have devised a computational method, based on multi-dimensional scaling, to decide upon the spatial arrangement of nanomagnets that matches a particular quadratic minimization problem. Each variable is represented by a nanomagnet and the distances between them are such that the dipolar interactions match the corresponding pairwise energy term in the original optimization problem. We select the nanomagnets that participate in a specific computation from a field of regularly placed nanomagnets. The nanomagnets that do not participate are deselected using transverse magnetic fields. We demonstrate these ideas by solving Landau-Lifshitz equations as implemented in the NIST's micro-magnetic OOMMF software.

I. INTRODUCTION

Driven by needs for very dense storage (HDD) and memory devices (MRAM) [1], there have been tremendous advances in fabrication methods for patterning magnetic media at nano-scales. Such magnets can be fairly well approximated by single magnetic domain models [2]. Moreover, by exploiting geometric and material anisotropy, one can construct nanomagnet that have two stable magnetic arrangements, which can be used to represent 0 and 1. The size of the nanomagnets in such devices is limited not by technology, but by the need to isolate the devices from each other, which is necessary for data storage. However, this dipolar interaction between neighboring magnets can be exploited for computing. So far, there have been suggestions for using them for Boolean logic based computing [3], [4]. Indeed, most work on nano-logic that seeks to replicate traditional computing involving logic and arithmetic operations [5], [6], [7], [8], [9]. We propose to harness the energy minimizing aspect of magnetic operations to solve directly quadratic optimization problems. We call this Magnetic Field-based Computing (MFC); it uses an interacting system of single-domain nanomagnets.

Since nano-devices are expected to have high error rates, both fabrication related and during operations, it makes sense to consider *error-tolerant applications*, where the cost of failure of not finding *the* optimal solution is not high; even

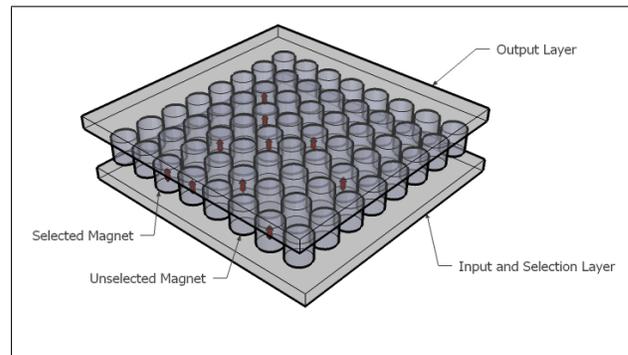


Fig. 1. Schematic of the hardware. A uniform 2D array of nanomagnets is sandwiched between two layers. One layer contains elements to select and provide input to each magnet and the other layer contains circuitry to read the magnetization of each cell. The activated or selected magnets participate in the computation.

solutions that are close to optimal ones suffices in practice. One such context is in quadratic optimization that arises in object recognition problems in computer vision. Unlike logic and arithmetic computing tasks that demand exact computations, vision problems can work with near optimal solutions. These vision problems place high demand on computational resources (on Boolean logic based computing platforms). Although, MFCs have not been used for image processing, arrays of quantum-dot (QD) arrays have been explored [10], [11] for low-level vision. However, mid-level vision tasks are very different in nature from image processing or low-level vision.

We use multi-dimensional scaling to decide upon the spatial arrangement of nanomagnets that matches a particular quadratic minimization problem. Each variable is represented by a nanomagnet and the distances between them are such that the dipolar interactions match the corresponding pairwise energy term in the original optimization problem. The arrangement of MFC cells that participate in the computation will be selected from a uniform array of MFC array, as shown in Fig. 1. Several techniques are available to achieve such regular patterning of nanomagnets [12], [13]. Each computation of energy minimization will involve selection of appropriate magnets from a regular arrangement, which, we show later, can be achieved using transverse magnetic fields.

II. THEORY: MFC ARRAYS AS ENERGY MINIMIZING ENGINES

The basic unit of computation is a nanomagnet with dimensions and materials such it exhibits single domain behavior. Material and geometric shape anisotropy can be exploited to orient the direction of the overall, ground state, least energy magnetization along a desired dimension. This direction and its opposite, which will also be another ground state, can be used to represent the logic states 0 and 1.

We can model the single domain behavior of each nanomagnet as a single ‘‘Heisenberg spin’’ vector, with its orientation as a continuous variable. The evolution of the magnetization of any (i -th) nanomagnet can be described by the Landau-Lifshitz-Gilbert equation of magnetic dynamics, containing reactive and dissipative terms.

$$\frac{d\mathbf{M}^{(i)}}{dt} = -\gamma\mathbf{M}^{(i)} \times \mathbf{B}^{(i)} - \frac{\alpha\gamma}{M_s}\mathbf{M}^{(i)} \times (\mathbf{M}^{(i)} \times \mathbf{B}^{(i)}) \quad (1)$$

where (i) $\mathbf{M}^{(i)}$ is the magnetization of the i -th magnet, (ii) γ is a precession coefficient, and (iii) α is the damping factor. The local effective field, $\mathbf{B}^{(i)}$, is given by $-\partial\mathcal{H}^{(i)}/\partial\mathbf{M}^{(i)}$, where $\mathcal{H}^{(i)}$ is the Hamiltonian component of the i -th magnet. The total Hamiltonian of an arrangement of magnets is the sum of these individual Hamiltonians and is given by

$$\mathcal{H} = \sum_i \mathbf{D}^i (\mathbf{M}^{(i)})^T \mathbf{M}^{(i)} + \sum_i (\mathbf{M}^{(i)})^T \mathbf{B}_{ext} + \sum_i \sum_{j \neq i} (\mathbf{M}^{(i)})^T \mathbf{C}(i, j) \mathbf{M}^{(j)} \quad (2)$$

where \mathbf{D}^i is the demagnetization tensor of the i -th magnet, capturing the shape anisotropy, $\mathbf{C}(i, j)$ is the interaction term between the i -th and j -th magnet, \mathbf{B}_{ext} is the external field. We have neglected the material anisotropy term as it is usually small, compared to the geometric anisotropy for elongated pillar magnets [14], the kind of magnets we are targeting in this study. At equilibrium, the value of $\mathbf{M}^{(i)}$, $i = 1, \dots, N$ minimizes the total Hamiltonian. For elongated, pillar, type magnet the magnetic vectors will be along the linear dimension. We can simplify the expression for the Hamiltonian by (i) choosing the z -axis to be along the linear dimension of the magnets, (ii) neglecting the x and y components of the geometry anisotropy and the magnetization, (iv) and consider the absence of external magnetic field.

$$\begin{aligned} \mathcal{H} &\approx \sum_i D_z^i (M_z^{(i)})^2 + \sum_i \sum_{j \neq i} M_z^{(i)} \mathbf{C}(i, j) M_z^{(j)} \\ &= \sum_i \sum_{j \neq i} M_z^{(i)} \mathbf{E}_{ij} M_z^{(j)} \end{aligned} \quad (3)$$

where we have collated all the z -components of the magnetizations into a vector, \mathbf{M}_z , and \mathbf{E} is the sum of the matrices composed out of the dipole-dipole interaction matrix, \mathbf{C} and D_z s, the geometric anisotropy term. It is a quadratic cost term involving pairwise ‘‘energies’’. The dipole-to-dipole interaction (energy) term is function of the distance between the two magnets on the 2D plane, d_{ij} , and given by [15].

$$\mathbf{E}_{ij} = \frac{\mu_0 |V|}{4\pi d_{ij}^3} (3d_{ij}^2 - 1) \quad (4)$$

where $|V|$ is a volume term that is constant for all magnets. This distance dependence of the interaction is what we will exploit. Any given arrangement of N pillar magnets can be taken to solve approximately a particular quadratic optimization problem with N variables, where the solution is specified by the z -component of the magnetization. Is it then possible to engineer the selection from a regular array of magnets so that it matches a quadratic optimization problem at hand? If so, then we can use the z -component of the magnetization as a solution, at least a good approximate solution. Before we go into details of how this is to be done, let us first consider a computationally expensive problem for which even such approximate solutions suffice.

III. THEORY: QUADRATIC PROBLEMS IN OBJECT RECOGNITION

The problem of object recognition involves the naming of objects found in an image. The solution involves (i) modeling of the objects of interest in term of features, such as corners, edges, and surfaces, (ii) extraction of corners, edges, and surfaces from the given image (segmentation), (iii) perceptual organization of these corners, edges, and surfaces into groups that most likely come form the object (grouping), and (iv) matching of the groups to the model. The third step of perceptual organization of the corners, edges, and surfaces separates the object features from the non-object features in the images. It is crucial in controlling the complexity of recognition. We will consider this particular application context for magnetic field computing (MFC) circuits.

Quadratic Energy Formulation: We represent a group or collection of features (edge segments) using the vector \mathbf{x} , whose i -th component, x_i , is 1 if the corresponding feature is part of the group and $x_i = 0$ if it is not. Between every pair of edge line we associate affinity energies, A_{ij} to capture the perceptual saliency of the relationship between them. For example, if two straight lines are parallel to each other then according to the Gestalt principles they are likely to belong to one object and hence the affinity should be high. Similarly, lines that are close together are more likely to be associated together, i.e. the proximity principle. Lines that form one straight arrangement are also likely to be grouped, i.e. the continuity principle. The quantitative forms of the affinity function vary in different implementation, but qualitatively they capture similar aspects. The particular form we use is

$$A_{ij} = \sqrt{l_i l_j} e^{-\frac{o_{ij}}{\max(l_i, l_j)}} e^{-\frac{d_{min}}{\max(l_i, l_j)}} \sin^2(2\theta_{ij}) \quad (5)$$

where l_i and l_j are the lengths of the i -th and j -th features, o_{ij} is the overlap, θ_{ij} is the angle, and d_{min} is the minimum distance between the two straight lines. We can see that this form of the affinity function agrees with the strengths of the Gestalt affinities that we want to model.

The goal is to find a group, \mathbf{x} , such that total affinity energy is maximized.

$$A = \sum_i \sum_{j \neq i} A_{ij} x_i x_j \quad (6)$$

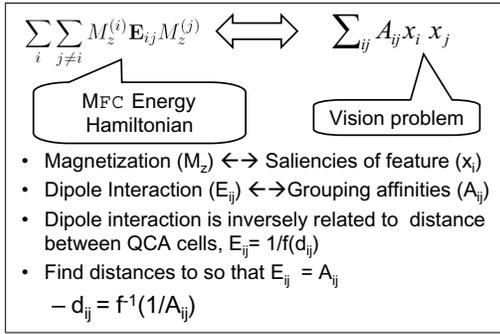


Fig. 2. Correspondence between the physics of interaction of a system of nanomagnets (MFC) and vision problem formulation as quadratic optimization and mapping of the entities involved.

IV. APPROACH: SYNTHESIS

Now we get back to the question: Is it then possible to select magnets from a regular array of magnets so that it matches a quadratic optimization problem at hand? Given pairwise perceptual affinities between N image features, how do we find placement in a two-dimensional space so that the distances between them would result in dipole interaction energies that is proportional to the given affinities? We have started to explore the use of the statistical method of multidimensional scaling [16], which can be used to embed distance matrices in low-dimension spaces and have so far been used extensively for data visualization.

Let N straight line have pairwise affinities $\{A_{ij}\}$. The goal of multidimensional scaling (MDS) is to find a configuration of points, representing these straight lines, in a $p = 2$ or 3 dimensional space such that the distance between two points i and j , denoted by d_{ij} , will be proportional to given affinities. If magnetic cells are placed at these point coordinates then the pairwise interaction between them will be proportional to the given energies, i.e. $E_{ij}^{MFC} \propto A_{ij}$. We proceed as follows.

Let the matrix Λ be constructed out of given affinities such that: $\Lambda_{rs} = \frac{1}{(A_{ij})^2}$. We desire to find the coordinate of each point in a p dimensional match, which we denote by the matrix of coordinate vector, $\mathbf{X}_{MDS} = [\mathbf{x}_1, \dots, \mathbf{x}_N]$, such that

$$(\mathbf{x}_i - \mathbf{x}_j)^T (\mathbf{x}_i - \mathbf{x}_j) = c \Lambda_{rs} \quad (7)$$

$$\mathbf{X}_{MDS}^T \mathbf{X}_{MDS} = -c \frac{1}{2} \mathbf{H} \Lambda \mathbf{H}, \quad \text{where } \mathbf{H} = (\mathbf{I} - \frac{1}{N} \bar{\mathbf{1}} \bar{\mathbf{1}}^T) \quad (8)$$

with \mathbf{I} as the identity matrix and $\bar{\mathbf{1}}$ as the vector of ones. This operator \mathbf{H} is referred to as the centering operator. These coordinates \mathbf{X} can be arrived at by classical MDS scheme [16]. The solution is based on the singular value decomposition of the centered distance matrix $\frac{1}{2} \mathbf{H} \Lambda \mathbf{H} = \mathbf{V}_{MDS} \Delta_{MDS} \mathbf{V}_{MDS}^T$ where \mathbf{V}_{MDS} , Δ_{MDS} are the eigenvectors and eigenvalues respectively. Assuming that centered distance matrix represents the inner product distances of a Euclidean distance matrix, the coordinates are given by

$$\mathbf{X}_{MDS} = (\mathbf{V}_{MDS} \Delta_{MDS}^{\frac{1}{2}})^T \quad (9)$$

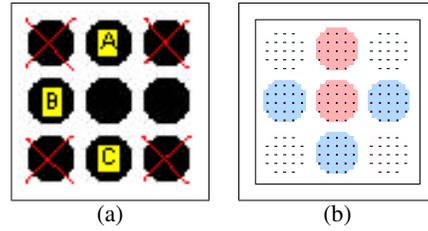


Fig. 3. Selection of cells using external electrical fields, simulated using OOMMF. (a) A 3 by 3 array of cells. We would like to de-select the cells marked with crosses and provide input to the cells marked with letters. (b) Final state of the magnetization along the dominant (easy) axis aligned along z -axis. The red and blue shadings denote the two states.

Note that we have dropped the constant of proportionality, c , since the energy minimizing solutions are invariant to scaling of the original function. Our nanomagnet selection solution is given by the first two rows of \mathbf{X}_{MDS} ; each column of this matrix gives us the coordinates of the corresponding nanomagnet to consider.

The computational overhead of this synthesis step is linear in the number of the image features. This replaces the complexity of the software solution to the minimization problem.

V. SELECTION OF NANOMAGNETS

We envision that the arrangement of MFC cells that perform in the computation will be selected from a uniform array of MFC array, as visualized in Fig. 1. However, how does one select the particular cells from an array of cells? We want to minimize the dipole interaction of the non-computing cells with other cells. Here we explore one possible idea. We could drive the non-computing cells into a state that is orthogonal to the computing state using an external field. To generate the needed magnetic field one can use current wire loops, such as those suggested by Csaba *et al.* [17]. If the ground states of the single domain magnets are along the z -axis, then drive the non-computing cells into state with magnetization along the xy plane. Some indication that this can work can be seen from our simulation experiments (using OOMMF [18]) shown in Fig. 3. We start with a 3 by 3 block of cylindrical magnets, shown in Fig. 3(a). The magnets that we want to de-select are the ones marked with a cross and the magnets for which we want to provide input are marked with a letter. Note that the selected cells form the majority logic structure. Fig. 3(b) shows the results for a magnet with uni-axial anisotropy along the z -axis. The non-computing cells are driven by an external field along the x -axis. The A, B, and C cells are driven by either up or down fields, denoting 1 or 0. We see that the output of structure does compute the majority logic.

A caveat we would like to add is that we are not claiming that we have a completely worked out a selection mechanism. This is just a sketch of one possible mechanism. Implementation level details will have to be worked out based on specific geometry, material, and interactions between magnets.

VI. RESULTS

We consider the specific version of the perceptual grouping problem of grouping straight line segments found in the edge images. One illustration of an input is shown in Fig. 4(a). Not that the three mutually parallel lines are perceptually salient i.e. they are the dominant structure that draws our attention.

For the arrangement of straight lines in Fig. 4(a) the embedding of the corresponding MFC pillar cells in a 2D space is shown in Fig. 4(b); if the MFC cells are placed at these locations, the dipole interactions will be proportional to the perceptual affinities between them. Fig. 4(c) shows the total magnetic field as computed by solving the Landau-Lifshitz-Gilbert equations using the OOMMF [18] package from NIST. These simulations were with Colbalt pillar nanomagnets with 10 nm diameter. Entire rectangular region was simulated using 225×195 , 1 nm cells, initialized with random magnetization. Fig. 4(d) shows the final magnetization of the pillars. The darkness of the intensity is proportional to the z -component of the magnetization. Note that the three pillars that represent the three parallel lines have the largest z -component. This three line group has been selected as the perceptually most salient.

VII. CONCLUSION

We have explored a new computing direction using nanomagnets. We considered directly solving quadratic optimization problems by harnessing the energy minimizing aspects of nanomagnet operations. In the past [19], we had shown how this type of computation can also be conducted using electronic quantum-dot cellular automata (e-QCA) by exploiting the electrical field interaction of electrons. Unlike QCA's, nanomagnets can operate in room temperatures [3]. We presented ideas, backed up by physics based simulation, to select nanomagnets from a regular array to participate in computation. We also demonstrated computation on a small vision problem to illustrate the feasibility of the approach. The attractive aspect of this kind of nano-computing is that is that the application itself is fault tolerant and the computations use a regular grid of nano devices, which is easier to fabricate than specific non-regular geometries.

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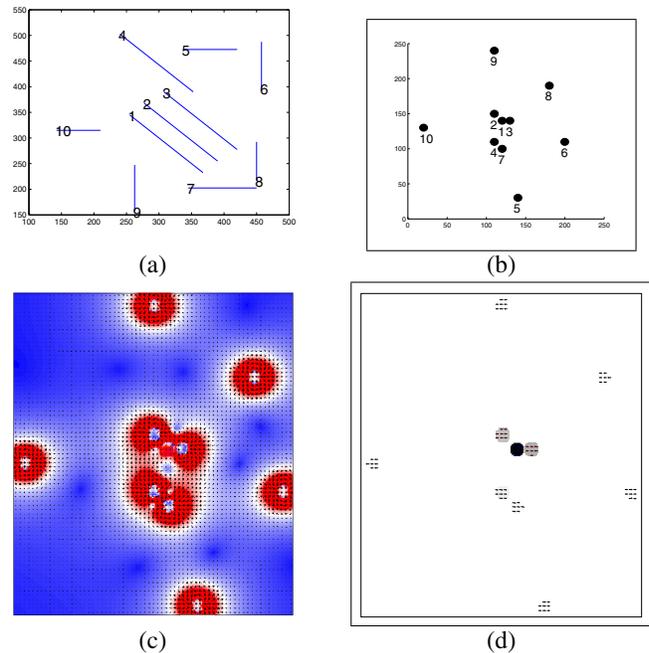


Fig. 4. Finding perceptually salient groupings of straight lines using nanomagnet arrays. (a) An arrangement of straight lines. The three lines in the center are salient. Each edge line in (a) is represented by a MFC magnet. (b) Top view of the spatial arrangement of cylindrical MFC magnets whose inter-distances are result in dipole moments that match the Gestalt affinities between the corresponding edge lines. (c) Total magnetic field among the pillar magnets, as computed by solving the Landau-Lifshitz equations using OOMMF [18]. (d) Magnetization of the pillar magnets. The darkness of the intensity is proportional to the magnetization along the z -direction. Note that the magnets representing the three most perceptually salient parallel straight lines in the middle are the have the largest values for M_z .

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