Nonlocal graph regularization for image colorization*

Olivier Lezoray, Vinh Thong Ta, Abderrahim Elmoataz
Université de Caen Basse-Normandie, GREYC UMR CNRS 6072
6 Bd. Maréchal Juin, F-14050 Caen, France
$\{$*olivier.lezoray, vinhthong.ta, abderrahim.elmoataz-billah*$\}$*@unicaen.fr*

Abstract

In this paper we present a simple colorization method that relies on nonlocal graph regularization. We introduce nonlocal discrete differential operators and a family of weighted p-Laplace operators. Then, p-Laplace regularization on weighted graphs problem is presented and the associated filter family. Image colorization is then considered as a graph regularization problem for a function mapping vertices to chrominances. Several results illustrate our framework and demonstrate the benefits of nonlocal graph regularization for image colorization.

1. Introduction

Colorization is the process of adding color to monochrome images. It is usually made by hand by color experts but this process is tedious and very time-consuming. In recent years, several methods have been proposed for colorization [4, 6] that less require intensive manual efforts. These techniques colorize the image based on the user's input color scribbles and are mainly based on a diffusion process. However, most of these diffusion processes only use local pixel interactions that cannot properly describe complex structures expressed by nonlocal interactions.

In this paper, we propose to use nonlocal graph regularization for image colorization. Regularization by variational methods has shown its effectiveness for many applications. Since the advent of the nonlocal means filters [1, 3], the use of nonlocal interaction to capture complex structures of the data has received a lot of attention. The nonlocal approach has also shown to be very effective and more flexible in the regularization process. We have recently proposed a nonlocal discrete p-Laplacian regularization framework for the processing of images and manifolds represented by weighted graphs of the arbitrary topologies [2]. The proposed methodology enables local and nonlocal regularization by using appropriated graphs topologies and edge weights. In this work, we consider the image colorization problem within this nonlocal graph-based framework.

*This work was supported under a research grant of the ANR Foundation (ANR-06-MDCA-008-01/FOGRIMMI)

2. Nonlocal graph regularization

In this Section, basic definitions on graphs are recalled, nonlocal discrete differential operators and a family of weighted p-Laplace operators are introduced. The p-Laplace regularization on weighted graphs problem is presented and the associated filter family.

2.1. Preliminary definitions

A weighted graph $G = (V, E, w)$ consists in a finite set $V = \{v_1, \ldots, v_N\}$ of N vertices and a finite set $E \subset V \times V$ of weighted edges. We assume G to be undirected, with no self-loops and no multiple edges. Let (u, v) be the edge of E that connects vertices u and v of V. Its weight, denoted by $w(u, v)$, represents the similarity between its vertices. Similarities are usually computed by using a positive symmetric function $w : V \times V \to \mathbb{R}_+$ satisfying $w(u, v) = 0$ if (u, v) is not an edge of E. The notation $u \sim v$ is also used to denote two adjacent vertices. Let $\mathcal{H}(V)$ be the Hilbert space of real-valued functions defined on the vertices of a graph. A function $f : V \to \mathbb{R}$ of $\mathcal{H}(V)$ assigns a real value $f(v)$ to each vertex $v \in V$. The function space $\mathcal{H}(V)$ is endowed with the usual inner product $\langle f, h\rangle_{\mathcal{H}(V)} = \sum_{v\in V} f(v)h(v)$, where $f, h : V \to \mathbb{R}$. Similarly, one can define $\mathcal{H}(E)$, the space of real-valued functions defined on the edges of G.

978-1-4244-2175-6/08/$25.00 ©2008 IEEE

2.2. Weighted gradient and divergence operators

Let $G = (V, E, w)$ be a weighted graph, and let $f : V \to \mathbb{R}$ be a function of $\mathcal{H}(V)$. The *difference operator* of f, noted $d : \mathcal{H}(V) \to \mathcal{H}(E)$, is defined on an edge $(u, v) \in E$ by: $(df)(u, v) = \sqrt{w(u, v)}(f(v) - f(u))$. The *directional derivative* (or *edge derivative*) of f, at a vertex $u \in V$, along an edge $e = (u, v)$, is defined as: $\partial_v f(u) = (df)(u, v)$. The *weighted gradient operator* of a function $f \in \mathcal{H}(V)$, at a vertex $u \in V$, is the vector operator defined by: $\nabla_w f(u) = [\partial_v f(u) : v \sim u]^T$. The $\mathcal{L}_2$-norm of this vector represents the *local variation* of the function f at a vertex of the graph. It is defined by $|\nabla_w f(u)| = \sqrt{\sum_{v \sim u} w(u, v)(f(v) - f(u))^2}$. The local variation is a semi-norm that measures the regularity of a function around a vertex of the graph. The *adjoint* of the difference operator, noted $d^* : \mathcal{H}(E) \to \mathcal{H}(V)$, is a linear operator defined by $\langle df, H \rangle_{\mathcal{H}(E)} = \langle f, d^* H \rangle_{\mathcal{H}(V)}$, for all $f \in \mathcal{H}(V)$ and all $H \in \mathcal{H}(E)$. Using the definitions of the inner products in $\mathcal{H}(V)$ and $\mathcal{H}(E)$, and of the difference operator, we obtain the expression of d^* at vertex u by $(d^* H)(u) = \sum_{v \sim u} \sqrt{w(u, v)}(H(v, u) - H(u, v))$. The *divergence operator*, defined by $-d^*$, measures the net outflow of a function of $\mathcal{H}(E)$, at each vertex of the graph.

2.3. A family of weighted p-Laplace operators

Let $p \in (0, +\infty)$ be a real number. The *weighted p-Laplace operator* of a function $f \in \mathcal{H}(V)$, noted $\Delta_w^p : \mathcal{H}(V) \to \mathcal{H}(V)$, is defined by $\Delta_w^p f(u) = \frac{1}{2} d^*(|\nabla_w f(u)|^{p-2} df(u, v))$. The p-Laplace operator of $f \in \mathcal{H}(V)$, at a vertex $u \in V$, can be computed by:

$$\Delta_w^p f(u) = \tfrac{1}{2} \sum_{v \sim u} \gamma_w^f(u, v)(f(u) - f(v)), \quad (1)$$

with $\gamma_w^f(u, v) = w(u, v)(|\nabla_w f(v)|^{p-2} + |\nabla_w f(u)|^{p-2})$. The p-Laplace operator is nonlinear, with the exception of $p = 2$. When $p = 2$, it corresponds to the *combinatorial graph Laplacian*. When $p = 1$, it corresponds to the *weighted curvature* of the function f on the graph.

2.4. p-Laplace regularization on weighted graphs

In this Section, one considers a general function $f^0 : V \subset \mathbb{R}^n \to \mathbb{R}^m$ defined on graphs of the arbitrary topologies and we want to regularize this function. The regularization of such a function corresponds to an optimization problem which can be formalized by the minimization of a weighted sum of two energy terms:

$$\min_{f \in \mathcal{H}(V)} \left\{ E_w^p(f, f^0, \lambda) = R_w^p(f) + \frac{\lambda}{2} \|f - f^0\|^2 \right\}. \quad (2)$$

with $R_w^p(f) = \frac{1}{p} \sum_{u \in V} |\nabla_w f(u)|^p$ a regularization functional. The first term in (2) is the smoothness term or regularizer, meanwhile the second is the fitting term. The parameter $\lambda \geq 0$ is a fidelity parameter, called the Lagrange multiplier, which specifies the trade-off between the two competing terms. Both terms of the energy E_w^p are strictly convex functions of f. In particular, by standard arguments in convex analysis, the problem (2) has a unique solution, for $p = 1$ and $p = 2$, which satisfies $\frac{\partial E_w^p(f, f^0, \lambda)}{\partial f(u)} = 0, \quad \forall u \in V$. This is rewritten as

$$\frac{\partial R_w^p(f)}{\partial f(u)} + \lambda(f(u) - f^0(u)) = 0, \quad \forall u \in V, \quad (3)$$

that is equivalent to:

$$2\Delta_w^p f(u) + \lambda(f(u) - f^0(u)) = 0, \quad \forall u \in V, \quad (4)$$

since $\frac{\partial R_w^p(f)}{\partial f(u)} = 2\Delta_w^p f(u)$, $\forall u \in V$. Substituting the expression of the p-Laplace operator in (3), we obtain:

$$\left(\lambda + \sum_{v \sim u} \gamma_w^f(u, v) \right) f(u) - \sum_{v \sim u} \gamma_w^f(u, v) f(v) = \lambda f^0(u). \quad (5)$$

We propose to use the linearized Gauss-Jacobi iterative method to solve the system (5). Let t be an iteration step, and let $f^{(t)}$ be the solution of (5) at the step t. Then, the method is given by the following algorithm $\forall u \in V$:

$$\begin{cases} f^{(0)}(u) = f^0(u) \\ f^{(t+1)}(u) = \dfrac{\lambda f^0(u) + \sum_{v \sim u} \gamma_w^{f^{(t)}}(u, v) f^{(t)}(v)}{\lambda + \sum_{v \sim u} \gamma_w^{f^{(t)}}(u, v)}. \end{cases} \quad (6)$$

It describes a family of discrete diffusion processes, which is parameterized by the structure of the graph (topology and weight function), the parameter p, and the parameter λ. Also, the stopping time can be given *a priori*, or can be determined by a stopping criterion. To get the convergence of the process, a classical stopping criterion is $\|f^{(t+1)} - f^{(t)}\| < \tau$, where $\tau \to 0$ is a small fixed constant. At each iteration of the algorithm (6), the new value $f^{(t+1)}(u)$ depends on two quantities: the original value $f^0(u)$, and a weighted average of the filtered values of $f^{(t)}$ in a neighborhood of u. This shows that the proposed filter, obtained by iterating (6), is a low-pass filter which can be adapted to many graph structures and weight functions.

Gray level image | Color scribbles | Local (21.66dB) | Nonlocal (21.75dB)

Figure 2. Colorization results in local or nonlocal configurations for a boy image.

2.5. Construction of graphs

The minimization problem (2), and the discrete diffusion processes (6), can be used to regularize any function defined on a finite set V of discrete data. This is achieved by constructing a weighted graph $G = (V, E, w)$, and by considering the function to be regularized as a function $f^0 : V \subset \mathbb{R}^n \to \mathbb{R}^m$, defined on the vertices of G. A typical graph in image processing is the 8-adjacency graph where vertices are associated to pixels and edges correspond to pixel adjacency relationships: this corresponds to a classical *local* processing. By changing the graph topology and the edge weights, we naturally obtain an expression of *nonlocal* processing methods which is embedded in the graph structure. Indeed, once edges are added between vertices, they are considered as direct neighbors and the processing is local over the graph. Figure 1 resumes all these elements. A classical 8-adjacency grid graph is considered and expresses only local interactions through local edges. An example of nonlocal interaction is depicted for one vertex by adding nonlocal edges between vertices not spatially connected (for a 5×5 neighborhood in Figure 1). Let $F(f^0, v) \in \mathbb{R}^q$ denote a feature vector associated to each vertex $v \in V$. This feature vector associated to vertices can be the initial function value: $F(f^0, v) = f^0(v)$ (for a *local* configuration) or a vector $F(f^0, v) = [f^0(u) : u \in B_{v,s}]^T$ (for a *nonlocal* configuration). For this latter case, $F(f^0, v)$ is a patch where $B_{v,s}$ denotes a bounding box of size $(2s+1) \times (2s+1)$ centered at v ($s = 1$ in Figure 1). Weights are computed according to a measure of similarity $w(u, v) = \exp\left(-\frac{\|F(f^0,u)-F(f^0,v)\|^2}{\sigma^2}\right)$. To have a parameterless weight function, σ has to be computed locally at each vertex of the graph [5].

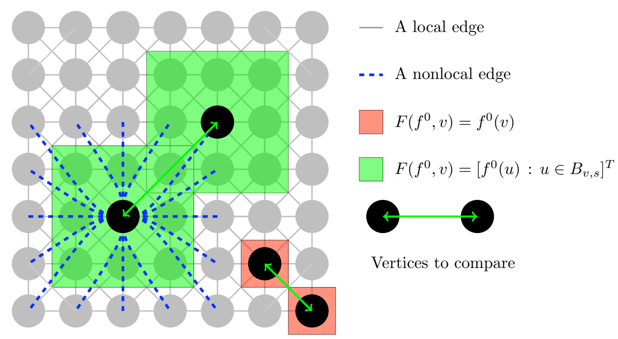

Figure 1. Graph topology and feature vector illustration.

3. Application to image colorization

We now explain how to perform image colorization with the proposed framework. From a gray level image $f^l : V \to \mathbb{R}$, a user provides an image of color scribbles $f^s : V_s \subset V \to \mathbb{R}^3$ that defines a mapping from vertices to a vector of RGB color channels: $f^s(v) = [f_1^s(v), f_2^s(v), f_3^s(v)]^T$ where $f_i^s : V \to \mathbb{R}$ is the i-th component of $f^s(v)$. From these functions, one computes $f^c : V \to \mathbb{R}^3$ that defines a mapping from the vertices to a vector of chrominances:

$$\begin{cases} f^c(v) = \left[\frac{f_1^s(v)}{f^l(v)}, \frac{f_2^s(v)}{f^l(v)}, \frac{f_3^s(v)}{f^l(v)}\right]^T, \forall v \in V_s. \\ f^c(v) = [0, 0, 0]^T, \forall v \notin V_s. \end{cases} \quad (7)$$

We then consider the regularization of function $f^c(v)$ by applying algorithm (6) where $\gamma_w^{f^{(t)}}(u, v)$ is replaced by $\gamma_w^{f^l}(u, v)$ and weights are computed on the gray level

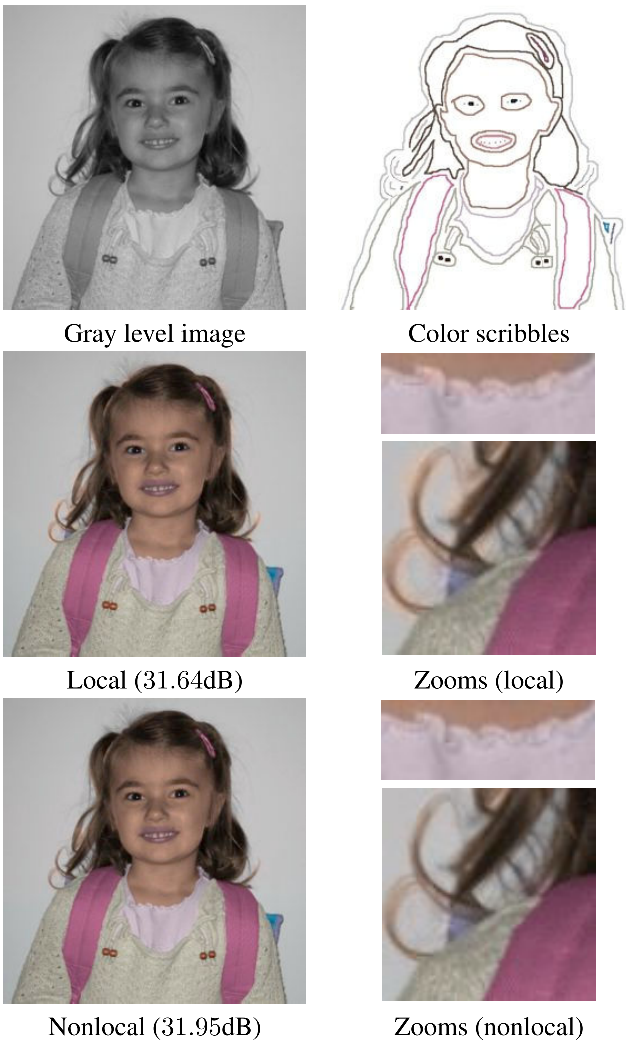

Figure 3. Colorization results in local or nonlocal configurations for a girl image.

image: $w(u,v) = \exp\left(-\frac{\|F(f^l,u)-F(f^l,v)\|^2}{\sigma_l^2}\right)$, with σ_l estimated from f^l. At convergence, final colors are obtained by $f^l(v) \times \left[f_1^{c^{(t)}}(v), f_2^{c^{(t)}}(v), f_3^{c^{(t)}}(v)\right]^T, \forall v \in V$. For all the experiments $\lambda = 0.01$, $p = 1$ and the data-fitting term is measured only on vertices $v \in V_s$. This is needed to ensure that the original scribbles do not change too much of color. Figures 2 and 3 present colorization results on two gray level images obtained from color images given a set of color scribbles as input. Local (8-adjacency grid graph with $F(f^l,v) = f^l(v)$) and nonlocal (99-adjacency grid graph that corresponds to add nonlocal edges in a 11×11 neighborhood with $F(f^l,v)$ as a 5×5 patch) are presented. The benefits of nonlocal processing are evident for the boy (Figure 2): the eyes and several areas of the bib are not properly colored and have diffused over straight edges. On the opposite, nonlocal colorization has successfully colored these areas thanks to its ability to discover similar textures and fine details. For the girl (Figure 3), even with precise locations of color scribbles, local results are less good than nonlocal ones: too much diffusion around the hairs and the shirt (see the zoomed areas in Figure 3). To have a quantitative evaluation between local and nonlocal colorization, PSNR measures between the original color image (before its conversion into a gray level one) and the colored one are provided and show once again the benefits of nonlocal colorization.

4. Conclusion

In this paper, we first presented a framework that enables local or nonlocal regularization by using appropriated graph topologies and edge weights. We have shown how the proposed framework can be used to perform nonlocal image colorization. Moreover, results were provided that demonstrate the benefits of nonlocal image colorization compared to local image colorization.

References

[1] A. Buades, B. Coll, and J.-M. Morel. A review of image denoising algorithms, with a new one. *Multiscale Modeling and Simulation*, 4(2):490–530, 2005.

[2] A. Elmoataz, O. Lezoray, and S. Bougleux. Nonlocal discrete regularization on weighted graphs: a framework for image and manifold processing. *IEEE transactions on Image Processing*, to appear, 2008,.

[3] G. Gilboa and S. Osher. Nonlocal linear image regularization and supervised segmentation. *SIAM Multiscale Modeling and Simulation*, 6(2):595–630, 2007.

[4] A. Levin, D. Lischinski, and Y. Weiss. Colorization using optimization. *ACM Trans. Graph.*, 23(3):689–694, 2004.

[5] O. Lezoray, S. Bougleux, and A. Elmoataz. Parameterless discrete regularization on graphs for color image filtering. In Springer, editor, *ICIAR*, Lecture Notes in Computer Science, pages 46–57, 2007.

[6] L. Yatziv and G. Sapiro. Fast image and video colorization using chrominance blending. *IEEE Transactions on Image Processing*, 15(5):1120–1129, 2006.