Modeling of Crack Depths in Digital Images of Concrete Pavements Using Optical Reflection Properties

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Abstract: Digital image-based automated pavement crack detection and classification technology has seen vast improvements in the recent years. Although crack lengths and widths can be evaluated using state-of-the-art software with a reasonable accuracy, no reported evidence is found in extending this technology to evaluate crack depths. As a supplement to the existing technology, additional information relevant to pavement crack severity could be revealed by the optical modeling of the image formation process and the subsequent analysis of the variation in pixel intensity profiles within images. A preliminary study was carried out to model the digital image formation of cracked concrete pavements based on the bidirectional reflection distribution function. This study was specifically focused on the optical modeling of shallow longitudinal and transverse cracks as well as joints of concrete pavements using the variation of reflection properties at surface discontinuities. Surface discontinuities were considered to be of regular geometrical shapes for simplification. The new image formation model revealed a definitive relationship among the crack widths and depths and the maximum pixel intensity contrasts seen in the images of the cracks. The model calibration involved the selection of reflection properties to match the pixel intensity contrasts across model generated images of cracks and joints against those of identical cracks formed in concrete pavements. The model predictions of crack depths were also verified using actual crack data not used in the calibration. Finally, the usefulness of the calibrated model in evaluating the depths of shallow cracks and differentiating cracks from joints and other surface irregularities in concrete pavements is illustrated.

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Introduction

Digital image-based automated pavement evaluation has been gradually replacing the manual pavement evaluation due to its improved efficiency and operational safety. At present, automated digital pavement image analysis is mostly focused on the detection and classification of pavement cracks (Ayenu-Prah and Attoh-Okine 2008; Chou et al. 1994; Huang and Xu 2006; Lee and Kim 2005; Liu et al. 2008; Wang 2000). Typical evaluation vehicles include an exterior line-scan camera that captures grayscale images of the pavement and a computer mounted inside the vehicle for acquisition, storage, and analysis of the captured images. The grayscale images are composed of individual pixels having intensity values in the range of 0 to 255 representing colors from black to white, respectively. A lighting system attached to the rear bumper of the survey vehicle provides adequate illumination for acquisition of images irrespective of natural lighting. In more recently developed imaging vehicles, the lamp-based artificial illumination has been replaced by laser lights to overcome the issues of nonuniform illumination and shadows (National Optical Institute 2008).

When digital pavement images are processed, a pixel intensity contrast is observed at cracks with the intensities inside the crack being significantly lower compared to the outside, if the cracks are not filled with sand or clay. The consequent color contrast is exploited in automated state-of-the-art pavement evaluation software to identify cracks. The automated assessment of the extent and severity of cracks based on the respective evaluation of crack lengths and widths becomes a useful input to pavement condition evaluation. On the other hand, an assessment of crack depths would be useful in determining rehabilitation strategies. Furthermore, when it is required to identify milling depths for asphalt pavement resurfacing projects, engineers depend on pavement core samples.

In addition, an evaluation of the depths of defects up to about 1 cm would be useful in distinguishing cracks from more superficial cracklike features that frequently appear in digital images of open-graded friction courses and lightly spalled concrete pavements. Therefore, a nondestructive means of evaluating even shallow crack depths would be invaluable in pavement evaluation and rehabilitation decision-making.

Although the pixel intensity variation within cracks is determined by the reflection characteristics and geometry of cracks, it has not been correlated to crack depths. Thus, digital image-based evaluations reach well short of the evaluation of the depths of cracks. The writers believe that if the image formation process is
modeled using appropriate optics it would lead to the revelation of even more useful information such as shallow crack depths from the digital images. The writers were unable to locate any published literature where attempts have been made to model the formation of digital images of cracked or jointed pavements using the principles of optics. Hence the objectives of the study documented in this paper are to model the intensity contrast at cracks and joints using the reflection properties of the intact pavement surface and cracks and use the model to predict the depths of cracks, joints, and other irregular features to distinguish each feature.

Modeling of Pavement Surface Radiance

Reflection Properties of Surfaces

The light incident on a surface could reflect completely in the opposite direction as dictated by the law of reflection or scatter in many directions. Surfaces where light reflection is solely in the opposite direction are known as specular surfaces while diffusive surfaces reflect light in all directions including the specular direction. Generally most surfaces have both specular and diffusive properties reflecting light in all directions but more so in the specular direction. The reflection properties of a pavement surface depend on the texture of the surface determined by the constituents of the pavement mix and the surface geometry. Hence the reflection properties can be used to model the unique characteristics of pavement surfaces including their distress features.

BRDF

The intensity of light reflection (radiance) from any point on a surface in the direction of the camera depends on the intensity of incident light (irradiance), the local reflectance properties, and the orientation of the surface at that point to the direction of incidence. The complex relationship between the radiance and irradiance of a surface can be best described by the bidirectional reflection distribution function (BRDF). When the intensity and direction of the radiant light are known the BRDF of the surface can be used to determine the intensity of the light that reflects from the surface in any given direction. The BRDF is defined as the ratio of radiance in a given direction (\( \tilde{R} \)) to the irradiance on that surface from another direction (\( \tilde{I} \)) (Ward 1992) [Eq. (1a)]

\[
BRDF(\tilde{I}, \tilde{R}) = \frac{L_r(\tilde{R})}{\tilde{L}_i(\tilde{I}) \cos \theta_i d\omega}
\]

where \( L_r(\tilde{R}) \) = radiance (reflected flux per unit normal area per unit solid angle); \( \tilde{L}_i(\tilde{I}) \) = irradiance (incident flux per unit normal area per unit solid angle); \( \theta_i \) = polar angle between the incident vector and the surface normal; \( d\omega \) = solid angle subtended at the surface point by the incident light source.

The total radiance at a point on the surface in the \( \tilde{R} \) direction due to all the light entering the hemisphere encompassing a solid angle of \( 2\pi \) steradian (sr) surrounding that point can be expressed as

\[
L_r(\tilde{R}) = \int_0^{2\pi} \int_0^\pi \rho(\tilde{I}, \tilde{R}) \tilde{L}_i(\tilde{I}) \cos \theta_i d\theta_i d\phi_i
\]

Therefore, in 3D, the above relationship is based on two incident and two reflected angles defining the directions \( \tilde{I} \) and \( \tilde{R} \) as expressed in Eq. (2)

\[
L_r(\theta_i, \phi_i) = \int_0^{2\pi} \int_0^{\pi/2} \rho(\theta_i, \phi_i; \theta_r, \phi_r) \tilde{L}_i(\theta_r, \phi_r) \cos \theta_i \sin \theta_i d\theta_i d\phi_i
\]

where \( \phi_i \) = azimuth angle of the incident vector projected onto the surface plane; \( \theta_i \) = polar angle between the reflected vector and the surface normal; \( \phi_r \) = azimuth angle of the reflected vector projected onto the surface plane; \( \tilde{L}_i(\theta_r, \phi_r) \) = reflected radiance (W/sr/m²); \( \rho(\theta_i, \phi_i; \theta_r, \phi_r) \) = BRDF (m²) given by Eq. (1).

The most common functions that are used to represent the BRDF of a surface are the tensor products of the spherical harmonics, Zernike polynomials, and spherical wavelets (Rusinkiewicz 1998). However, most of the basic functions require a large number of coefficients to describe even moderately specular BRDFs. In addition, the above methods do not require any less storage even under isotropic BRDF conditions. Rusinkiewicz (1998) proposed a method for decomposing BRDFs by changing variables more efficiently. In the Rusinkiewicz (1998) transformation, the BRDF is represented as a half angle between the incident and reflection directions, and the difference angle between the half angle and incident angle (\( \hat{h} \) in Fig. 1). Kautz and McCool (1999) used single value decomposition (SVD) and normalized decomposition (ND) for the BRDF function. On the other hand, simplified models are also available for approximate evaluation of the BRDF such as the models of Phong (1975), He et al. (1991), Cook and Torrance (1981), and Ward (1992).

Ward’s Reflection Model for BRDF

In the work reported here, Ward’s reflection model (1992) is used due to its simplicity and physically meaningful parameters. Although Ward (1992) formulated reflection models for both anisotropic and isotropic surfaces, in this preliminary analysis performed by the writers the isotropic model given by Eq. (3) has been assumed for simplicity.
where \( \rho_d \) = diffuse reflectance (sr\(^{-1} \)); \( \rho_s \) = specular reflectance (sr\(^{-1} \)); \( \delta \) = angle between vectors \( \hat{n} \) and \( \hat{h} \) in Fig. 1 [\( \hat{n} \) = unit normal to the surface; \( \hat{h} \) = half vector between the unit incident vector \( \hat{d} \) and unit reflection vector \( \hat{d}' \) as illustrated and defined in Eq. (4)]: 
\[ \alpha = \text{standard deviation of the surface slope or the square root of the slope variance.} \]

**Selection of Surface Parameters for Ward’s Model**

The three dominant parameters in the Ward’s reflection model [Eq. (3)] are \( \rho_d, \rho_s \) and \( \alpha \). The parameter \( \alpha \) represents the randomness in the orientation of the tiny fractals that form the surface. Typically surfaces with relatively lower \( \alpha \) values can be considered specular because smooth surfaces that are characterized by low slope variances (\( \alpha \)) reflect light mostly in the specular direction with much lower reflection components in the other directions. As \( \alpha \) increases, more and more light reflects in the directions other than the specular direction and the surface assumes diffusive characteristics. Thus a very low but a nonsingular \( \alpha \) value such as 0.0001 defines a surface with high specularity for all practical purposes, where as a relatively high \( \alpha \) value of 0.2 portrays a highly random fractal orientation and hence a diffusive surface. However, once a limiting high value of \( \alpha \) is reached, one can expect the reflection to reduce in all directions due to obscurity and internal reflection caused by the interference from the surface profile itself. Although the parameter \( \alpha \) could depend on both the macrotexture and the microtexture of a pavement, it is more sensitive to the microtexture.

On the other hand, the parameters \( \rho_d \) and \( \rho_s \) determine the respective magnitudes of the total specularity and the total diffusivity that are inherent to that surface. As seen in Eq. (3) the specular component is assumed to be a Gaussian distribution with its standard deviation related to \( \alpha \). Typical values of \( \rho_d \) and \( \rho_s \) for different surfaces are reported in Ward (1992). However, the specific values of \( \rho_d, \rho_s, \) and \( \alpha \) in the writers’ model are estimated based on comparing the model predictions to corresponding experimental data. Gonioreflectometers are commonly used to measure the BRDF of a given surface. It is recommended to use at least 2,500 data points to sample the BRDF at every 10\(^o\) on isotropic surfaces and at least 100,000 data points to sample the BRDF on anisotropic surfaces (Joshi et al. 2007). Marschner et al. (2000) presented an image-based process for measuring the BRDF of a surface with an apparatus consisting of two cameras, a light source, and a test sample of known shape.

**Modeling of the Image Intensity Variation across Cracks**

For modeling purposes a crack can be considered simply as a discontinuity with a V-shaped (triangular) cross section on a homogeneous pavement surface. In the following formulation the writers demonstrate how the contrast and the pixel intensity variation across the image of a pavement discontinuity can be modeled. In this formulation the light source attached to the rear bumper of the imaging vehicle is considered as a transverse line light source that provides uniform and continuous illumination. As the vehicle moves forward, the line-scan camera scans the pavement in a plane perpendicular to the direction of travel capturing the image of a strip of the entire width of the pavement at a given instant. This line strip is considered to be one pixel in width and completely illuminated by the above light source at the image capturing instant. As shown in Figs. 2, 3, and 5, at any instant, the camera (\( C \)) only points to a single point (\( P \)) of the pavement when imaging the pavement. Hence it can be assumed that the pixel intensity corresponding to the image of \( P \) is due to the reflection from the incident illumination at \( P \) from the entire light source (\( L \)). Other assumptions made in this formulation are that (1) the heights of the light source and the camera are relatively larger than the crack depths; and (2) two cameras are used with each one pivoted immediately above each wheel path. These two assumptions ensure the maximum contrast to be obtained in the images of cracks and preclude the need to consider inter-reflection within the cracks.

**Case 1—Modeling the BRDF of Longitudinal Cracks**

For this case, as shown in Fig. 2, the analysis can be performed in 2D assuming the cross section of the crack to be invariant in the \( z \) (longitudinal) direction. Hence the \( z \) (\( \hat{k} \)) axis that comes out of the paper is not shown in Fig. 2. The following vectors can be defined at any point \( P(x, y) \) with respect to the illumination due...
to an element of length $\delta x^*$ of the light source at $L(lx, h)$, where $h$ = height of the light source above the pavement.

incident vector, $\vec{I} = (px - lx)\hat{i} + (py - l)\hat{j}$  

reflected vector, $\vec{R} = -(px)\hat{i} + (cy - py)\hat{j}$  

The following relationships are valid in the respective domains:

Outside the crack

$$py = 0$$  

unit normal to the pavement surface, $\vec{n} = \hat{j}$

Inside the crack

For $|x_1| < |px| < |x_1 + a/2|

$$py = -2 \left( \frac{d}{a} \right) (px - x_1)$$  

unit normal, $\vec{n} = \frac{d}{\sqrt{d^2 + a^2/4}}\hat{i} + \frac{a/2}{\sqrt{d^2 + a^2/4}}\hat{j}$

For $|x_1 + a/2| < |px| < |x_1 + a|

$$py = -2 \left( \frac{d}{a} \right) (|x_1| + a - |px|)$$  

unit normal, $\vec{n} = -\frac{d}{\sqrt{d^2 + a^2/4}}\hat{i} + \frac{a/2}{\sqrt{d^2 + a^2/4}}\hat{j}$

The above results can be used to obtain the variables $\theta_r$, $\theta_i$, and $\delta$ [Eq. (3)] for Case 1 as shown in the section on converting the BRDF to pavement surface radiance [Eqs. (15a)–(15c)].

**Case 2—Modeling the BRDF of Transverse Cracks**

Since the geometry of a transverse crack (Fig. 3) varies along the transverse planes perpendicular to the travel direction where both illumination and imaging occur, variations along the longitudinal direction ($z$) (Fig. 4) must be considered in this analysis.

The following vectors can be defined at any point $P(px, py, pz)$ with respect to illumination due to an element of length $\delta x^*$ of the light source at $L(lx, h)$

incident vector, $\vec{I} = (px - lx)\hat{i} + (py - l)\hat{j} + (pz)\hat{k}$  

reflected vector, $\vec{R} = -(px)\hat{i} + (cy - py)\hat{j} + (pz)\hat{k}$

The following relationships are valid in the respective domains:

Outside the crack

$$py = 0$$

unit normal to the pavement surface, $\vec{n} = \hat{j}$

**Case 3—Modeling the BRDF of Joints**

The following vectors can be defined in Fig. 5 at any point $P(px, py, pz)$ with respect to illumination due to an element of length $\delta x^*$ of the light source at $L(lx, h)$

incident vector, $\vec{I} = (px - lx)\hat{i} + (py - l)\hat{j}$  

reflected vector, $\vec{R} = -(px)\hat{i} + (cy - py)\hat{j}$

The following relationships are valid in the respective domains:

Inside the crack for $0 < pz < a/2$

$$py = -2 \left( \frac{d}{a} \right) pz$$

$$\vec{n} = \frac{a^2}{\sqrt{d^2 + a^2/4}}\hat{j} - \frac{d}{\sqrt{d^2 + a^2/4}}\hat{k}$$

For $a/2 < pz < a$

$$py = -2 \left( \frac{d}{a} \right) (a - z)$$

$$\vec{n} = \frac{a^2}{\sqrt{d^2 + a^2/4}}\hat{j} + \frac{d}{\sqrt{d^2 + a^2/4}}\hat{k}$$

The above results can be used to obtain the variables $\theta_r$, $\theta_i$, and $\delta$ [Eq. (3)] for Case 2 as shown in the section on converting the BRDF to pavement surface radiance [Eqs. (15a)–(15c)]. Furthermore, in both Cases 1 and 2, provisions were made in the computer code BRDFimage that programmed the above analysis to account for the possibility of obscurity of some locations inside cracks with respect to the camera and the light source.

**Inside the crack:**

$$py = 0$$

unit normal to the pavement surface, $\vec{n} = \hat{j}$
Conversion of BRDF to Pavement Surface Radiance

For all the above Cases (1)–(3), in relation to Ward’s model [Eq. (3)]

\[
\cos \theta_{L} = -\vec{I} \cdot \vec{\hat{n}} \quad \text{(15a)}
\]

\[
\cos \theta_{r} = \vec{R} \cdot \vec{\hat{n}} \quad \text{(15b)}
\]

\[
\cos \delta = (1 - \vec{I} \cdot \vec{\hat{n}}) \cdot |\vec{I} + \vec{R}| \quad \text{(15c)}
\]

Therefore, the BRDF \( \rho(px,py,pz) \) on the pavement surface and the crack surfaces can be evaluated by substituting from Eq. (15) in Eq. (3). Finally, the radiance of \( P \) can be determined from Eq. (1b) using the following additional relationships. It can be seen from Fig. 6 that the solid angle subtended at \( P \) by the light source element \( \delta x' \) at \( L(x,h) \) is given by (normal component of area/square of distance)

\[
\delta O = \frac{[b(d\delta x')]\cos \theta_{L}}{D^{2}} \quad (16a)
\]

\[
\cos \theta_{L} = \vec{I} \cdot \vec{\hat{k}} \quad (16b)
\]

where \( \theta_{L} = \text{angle between the line light source and the normal to the incident light vector; } \delta x' = \text{considered length of the light source element at } L(x,h); D = \text{distance } LP; b = \text{width of line light source.} \)

If the light intensity attenuation is assumed to be inversely proportional to the distance, then from Fig. 6 it can also be seen that

\[
\frac{L(\vec{I})}{L^{*} = \frac{h^{2}L^{*}}{D^{2}}} \quad (16b)
\]

\[
L^{*} = \text{irradiance at the point } Q \text{ on the pavement immediately underneath the light source. Finally, by substituting from Eqs. (16a) and (16b) in Eq. (1b)}
\]

\[
L_{R}(px,py,pz) = \int_{-l/2}^{l/2} \rho(px,py,pz,x') \cdot \frac{h^{2}L^{*} \cos \theta_{L}}{D^{2}} \cdot \frac{[b(d\delta x')] \cos \theta_{L}}{D^{2}} \quad (16c)
\]

where \( l = \text{total length of the light source.} \)

Conversion of Pavement Radiance to Image Intensities

The image of a pavement feature is formed when the light reflected from that feature (object) enters the camera lens (aperture) and is refracted to the CCD sensor. A schematic diagram of the optics of image formation is shown in Fig. 6. The relationship between the radiance from any given object on the surface of the pavement and the intensity of the image can be expressed using the following formulation.

The parameters \( f, d_{c}, \) and \( D_{c} = \text{focal length, aperture diameter, and the distance between the camera center and the imaged point } (P), \) respectively. \( \delta O = \text{area surrounding } P \text{ from which light is reflected to the corresponding area } \delta I \text{ of the image.} \)

The flux leaving the pavement surface area \( \delta O \) toward the camera is given by

\[
F = L_{R} \delta O \delta \Omega = \frac{\lambda \cos \theta_{r}}{D_{C}^{2}} \quad (17a)
\]

where \( L_{R} = \text{radiance at } \delta O \)

\[
\delta \Omega = A_{C} \cos \theta_{r} \quad (18)
\]

\( A_{C} = \text{area of the camera and } \delta O = |\vec{R}| \) [from Eq. (5b), Eq. (9b), or Eq. (13b)]

If it is assumed that there is no loss of photo energy at the camera and the intensity at the image point \( \delta I \) is \( E, \) then

\[
F = E \cdot \delta I \quad (19a)
\]

Substituting from Eq. (19a) in Eq. (17)

\[
E \delta I = L_{R} \delta O \delta \Omega \quad (19b)
\]

Using Eq. (18), Eq. (19b) can be rewritten as

\[
E \delta I = L_{R}(\delta O)A_{C} \cos \theta_{r} \quad (19c)
\]

From basic optics it follows that the light originating from the object \( \delta O \) and passing through the center of the camera lens continues without refraction to form the image \( \delta I. \)

Hence the solid angles subtended at the camera center \( O \) by \( \delta O \) and \( \delta I \) are equal and opposite. Then by evaluating each solid angle one obtains

\[
\frac{(\delta O) \cos \theta_{r}}{D_{C}^{2}} = \frac{(\delta I) \cos \theta_{C}}{(f \cos \theta_{C})^{2}} \quad (19c)
\]

or

\[
\frac{\delta O}{\delta I} = \frac{\cos^{3} \theta_{C} \cdot D_{C}^{2}}{f^{2} \cos \theta_{C}} \quad (20)
\]

where \( \theta_{C} = \text{angle between the normal to the aperture of the camera and the reflected vector from point } P. \) Substituting in Eq. (19c) from Eq. (20)

\[
E = L_{R} \cdot \frac{\cos^{3} \theta_{C}}{f^{2}} \cdot A_{C} \quad (21)
\]

For a line-scan camera since \( \theta_{C} \) is equal to zero because the camera is oriented toward the imaging location and \( A_{C}, \) and \( f \) are constants, the intensity \( E \) of the image corresponding to the sur-
face point P is proportional to the radiance of P (or $L_P$). Hence it can be deduced that Eq. (16c) itself can be used conveniently to model the relative pixel intensities in a pavement image.

### Analysis of Model Predictions

In the theoretical simulation of the image formation, the line light source and the camera were assumed to be 4 ft and 9 in., respectively, above the pavement surface. The widths (a) and depths (d) of the modeled cracks were selected in the ranges of 1–10 mm and 0.5–20 mm, respectively, encompassing the ranges of corresponding values for most shallow cracks found in concrete pavements. Typical $\rho_{p}$, $\rho_{d}$, and $\alpha$ values (Ward 1992) were used in the reflection model for the intact pavement surface as well as cracks and joints. The relatively high roughness inside cracks and joints due to the high variability in the orientations of surface fractals requires higher $\rho_{p}$ and $\alpha$ values to be assumed inside cracks and joints compared to the respective values of the neighboring pavement surface areas. The main objective of the image model calibration is to establish two specific average sets of values for the parameters $\rho_{p}$, $\rho_{d}$, and $\alpha$ for a given pavement surface and a considered type of shallow cracks on it.

For a given crack width, as the depth of the crack increases the illumination of the tip of the crack decreases due to a lower amount of light reaching it and hence the minimum intensity within the crack, which occurs at the tip, decreases. This results in an increased image pixel intensity reduction (contrast) between the outside and the tip of the crack for higher crack depths. The formulated model can be used to observe the variation of the pixel intensity reduction from the outside to the tip of the crack (i.e., the maximum pixel intensity drop in the crack) with the depth for different crack widths. For two selected sets of $\rho_{p}$, $\rho_{d}$, and $\alpha$ values from Table 1, (0.6, 0.4, and 0.1) for the uncracked surface and (0.8, 0.2, and 0.2) for the surface inside a longitudinal crack, this variation is shown in Fig. 7. During the simulation the intensity outside the crack is assumed to be 170 in the grayscale range of 0–255 based on average measurements of intensities on the pavement surface at the given lighting conditions. It can be observed from Fig. 7 that for all crack widths ranging from 1 to 10 mm, the reduction in intensity reaches a limiting value when the depth of the crack tip is approximately twice the width of the crack. It must be noted that, theoretically, the limiting value of the maximum reduction in pixel intensity ($\Delta I_{\text{max ult}}$) must be equal to the pixel intensity of the uncracked surface (170 in this case) corresponding to the scenario of zero pixel intensity at the crack tip. However, the family of curves in Fig. 7 reaches limiting values between 140 and 160 at finite crack depths.

It can also be noted in Fig. 7 that the initial slope $[f(a)]$ depends on the width of the crack (a). Therefore, in order to develop an equation for the family of curves shown in Fig. 7 the relationship between the initial slope and the crack width can be expressed by Eq. (22)

$$f(a) = ka^m$$

where $k$ and $m$ are parameters governed by the optical properties of $\rho_{p}$, $\rho_{d}$, and $\alpha$ of the intact surface and the cracks. The logarithmic form of Eq. (22) can be expressed by Eq. (23)

$$\ln(f(a)) = \ln(k) + m \ln(a)$$

Fig. 8 shows the plot of Eq. (23) corresponding to data in Fig. 7. One remarkable finding of the analysis is that the selected format [Eq. (22)] to express the initial slope of such families of curves for both longitudinal and transverse cracks consistently produced high $R^2$ values for any selected set of $\rho_{p}$, $\rho_{d}$, and $\alpha$ values. When the values of $k$ and $m$ are determined from the intercept and slope of the fitted line in Fig. 7, Eq. (22) can be rewritten as

$$f(a) = 366a^{-0.97}$$

When the initial slopes are expressed analytically [Eq. (22)] the family of curves in Fig. 7 can be modeled using the following hyperbolic relationship based on the constant ultimate intensity difference ($\Delta I_{\text{max ult}}$) reached by the curves:

$$\ln(f(a)) = -0.9653a + 5.9025$$

$R^2 = 0.9674$
The above analytical development with respect to Figs. 7 and 9 shows that the maximum pixel intensity reduction in a longitudinal or transverse crack predicted by theoretical simulation can be given by

\[ \Delta I_{\text{max}} = \frac{d}{f(a)} + \frac{d}{(\Delta I_{\text{max}})_{\text{ult}}} \]  \hspace{1cm} (25)

where \( d \) = depth of the tip. Based on Fig. 7 and Eq. (24) the specific form of Eq. (25) for longitudinal cracks for the selected sets of \((p_d, p_s, \text{ and } \alpha)\) can be given by

\[ \Delta I_{\text{max}} = \frac{d^{0.97}}{366} + \frac{d}{(\Delta I_{\text{max}})_{\text{ult}}} \]  \hspace{1cm} (26)

A similar analytical procedure was adopted to model the variation of the maximum pixel intensity reduction in transverse cracks as well, with increasing crack depths. The family of curves (Fig. 9) predicted by the reflection model corresponding to the selected set of \( p_d, p_s, \text{ and } \alpha \) values can be expressed using the information in Fig. 10 as

\[ \Delta I_{\text{max}} = \frac{d^{1.27}}{106} + \frac{d}{[(\Delta I_{\text{max}})_{\text{ult}} - 55]} \]  \hspace{1cm} (27)

The above analytical development with respect to Figs. 7 and 9 shows that the maximum pixel intensity reduction in a longitudinal or transverse crack predicted by theoretical simulation can be expressed using a simple mathematical format.

![Fig. 9. Maximum pixel intensity reduction versus the depth for transverse cracks predicted by the reflection model](image)

![Fig. 10. Plot of \( \ln(f(a)) \) versus \( \ln(a) \) for transverse crack (uncracked area \( p_d=0.6, p_s=0.4, \text{ and } \alpha=0.1 \); inside crack \( p_d=0.9, p_s=0.1, \text{ and } \alpha=0.2 \)](image)

![Fig. 11. (a) Digital image of longitudinal concrete crack \( a = 5.4 \text{ mm}, d=2.5 \text{ mm}; \) (b) modeled digital image of the crack in Fig. 11(a)](image)

### Calibration of the Model

#### Experimental Setup

In order to calibrate the formulated reflection model (BRDFimage), the digital images of actual concrete pavement cracks with comparable dimensions were compared to the digital images created using the above model. For this purpose, a surface was concreted and V-shaped cracks of desired dimensions were formed on it under regulated conditions before the concrete hardened. The crack widths and depths in the ranges of 1–10 mm and 0.5–20 mm were selected to match the modeled cracks. Table 2 shows the locations of the camera and the light source with respect to the 3D coordinate axis system in Figs. 2, 3, and 5 in the experimental setup. In this study the optimum positions for the light source and the camera were selected (Table 2) to provide the highest contrast between the uncracked and cracked areas. When imaging a given crack it was placed at the origin of coordinates with the camera oriented toward it while the pavement surface was set to be in the \( x-z \) plane.

The crack widths were measured using a Vernier caliper while the crack depths were gauged using a thin wire. It must be noted that the cracks and joints had to be artificially formed in this preliminary phase of the research such that \( a/d \) ratios could be regulated well and uniform conditions could be achieved along the cracks and joints. Uniformity was further ensured by smoothing the concrete surface with a sand paper. A sample of captured digital images of a longitudinal crack, a transverse crack, and a joint are shown in Figs. 11(a), 12(a), and 13(a), respectively.

#### Results of Model Calibration

Predictions of the maximum pixel intensity reduction for a number of modeled longitudinal and transverse cracks were selected from Figs. 7 and 9 for comparison with those of the corresponding actual images. A sample of the modeled images is shown in Figs. 11(b), 12(b), and 13(b) against their experimental counter-
parts. In order to form a rational basis for comparison of the actual images and the corresponding modeled images, the intensities of the two sets of images (modeled and actual) were normalized with respect to the intensities of the immediate (uncracked) outside vicinity of the crack. Then the pixel intensity distributions within the digital images of actual cracks were compared to those of the modeled images.

The comparison of the distribution of intensities across experimental and modeled cracks for selected longitudinal and transverse cracks and a joint are shown in Figs. 14–16. From the plots in Figs. 14–16 it can be observed that the intensities inside actual cracks are distributed more randomly compared to those in the surrounding area. The general randomness is due to the nonhomogeneity of the concrete mix and one can expect this nonhomogeneity to be even more pronounced inside the cracks due to their irregular walls. In contrast, the pixel intensity variation predicted by the reflection model shows an abrupt decrease in pixel intensity up to the crack bottom [Figs. 14(b), 15(b), and 16(b)]. This is because one simplifying assumption made in the optical modeling is that both surfaces inside and outside the crack are individually homogeneous. As a result the intensity contrast at the boundaries of cracks and the joint of the modeled images are clearer than those of the corresponding images of the actual crack and the joint [Figs. 14(a), 15(a), and 16(a)]. Therefore, when the optimum optical parameters \( (\rho_c, \rho_n, \text{ and } \alpha) \) for the new model were determined during the calibration, only the minimum pixel intensity values of the actual cracks were chosen from Figs. 14(a), 15(a), and 16(a) for comparison with the intensities of the modeled images.

All in all, Figs. 14–16 show that despite the randomness in the intensity profile in actual cracks and joints due to the inherent
nonhomogeneity, the intensity profiles in the images of cracks and joints can be simulated reasonably well based on the calibrated reflection model. Similarly, the parameters needed in the new model to predict the intensity variation of joints were also estimated using the intensity distributions across joints. Based on the above described calibration the optimum values of $\rho_d$, $\rho_i$, and $\alpha$ obtained for the three cases considered are given in Table 3. As seen in Table 3 it was observed that the model could be calibrated with invariant reflection properties both outside and inside the cracks and joints.

### Verification of the Model

The predictive Eqs. (26) and (27) were verified by plotting the maximum intensity reductions and depths of additional experimental cracks that were not used in the calibration procedure, on the plots generated by Eqs. (26) and (27). For this exercise, based on the definition of $\Delta I_{\text{max}}$ in Eqs. (26) and (27), it would be replaced by the pixel intensity of the uncracked pavement that can be extracted from the images of the experimental cracks. Knowing $\Delta I_{\text{max}}$ for a crack with a known width, the corresponding prediction curve can be generated using Eqs. (26) and (27). While the curves in Figs. 17(a and b) and Figs. 18(a and b) illustrate the respective predictions of Eqs. (26) and (27) for longitudinal and transverse cracks, the discrete points on the same figures show the data obtained from the experimental cracks. It was seen that 80% of the overall predictions yielded errors less than 50% and 52% of the predictions yielded errors lower than 30%. The error was defined as the difference between the actual and predicted depths over the actual depth of cracks. The errors were seen to be more significant for shallow depths possibly indicating that the main source of the errors could have been the estimation of the widths and actual depths of cracks using manual methods.

Furthermore, Figs. 19(a and b) illustrate the correlation between the model predicted crack depths and the measured crack depths. From Figs. 19(a and b) it is seen that there is a satisfactory correlation (high $R^2$) between the predicted crack depths and the actual ones with about 0.7 mm of systematic underprediction. However, these results can be improved by modeling the random variations of the image intensity in the uncracked and cracked areas. As seen in Figs. 14(a), 15(a), and 16(a), the random variations in the actual images certainly affect the accurate determina-

### Table 3. Estimated Parameters for the Model

<table>
<thead>
<tr>
<th>Type of feature</th>
<th>Location</th>
<th>$\rho_d$</th>
<th>$\rho_i$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal crack</td>
<td>Inside</td>
<td>0.8</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>Outside</td>
<td>0.6</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>Transverse crack</td>
<td>Inside</td>
<td>0.9</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>Outside</td>
<td>0.6</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>Joint</td>
<td>Inside</td>
<td>0.9</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>Outside</td>
<td>0.6</td>
<td>0.4</td>
<td>0.1</td>
</tr>
</tbody>
</table>

**Fig. 16.** (a) Intensity distribution across concrete joint (1 pixel=0.04 mm); (b) intensity distribution across modeled joint of Fig. 13(a) (1 pixel=0.04 mm)

**Fig. 17.** (a) Predicted and measured maximum intensity reductions in experimentally created longitudinal cracks (widths lower than 5 mm); (b) predicted and measured maximum intensity reductions in experimentally created longitudinal cracks (widths larger than 5 mm)
of the maximum intensity contrast in particular and hence the model predicted crack depths. The observed extent of matching between the model predictions and the actual data is encouraging in view of the simplifying assumptions made in the model especially with respect to homogeneity, isotropy, and the regular geometry of cracks.

One realizes that the intensity contrast in the image of a joint is either due to the difference in reflection properties between the filler material and the pavement or the abrupt 90° surface slope change that occurs in the case of joints with missing fillers. Hence the images of joints would exhibit a high gradient of pixel intensity variation or rate of change of intensity per pixel. The above fact becomes evident from the comparison of Figs. 14 and 15(a) with Fig. 16(a). Therefore, cracks can be differentiated from joints based on the gradient of the pixel intensity variation.

**Potential Application in Pavement Rehabilitation**

Most sealing and repair strategies of cracked concrete pavements depend on the depths of cracks. Also, in the case of cracked asphalt pavements a knowledge of crack depths is essential to determine milling depths prior to resurfacing. At present, the milling depths are determined by the 95th percentile of the crack depth distribution established by three core samples retrieved from the existing pavement. Currently no nondestructive evaluation method is available to determine even shallow crack depths in an entire pavement network in a speedy manner. Hence evaluation of approximate crack depths based on the processing of crack images would be invaluable to pavement maintenance decision making.

Furthermore, when state-of-the-art crack identification software are used to evaluate cracks in superpave pavements topped with open-graded friction courses, some surface irregularities can be misidentified as small cracks. Even in the case of concrete pavement evaluation, spalling and other surface defects can interfere with crack identification. Therefore, the procedure formulated in this paper for identification of shallow crack depths up to 1 cm would be useful in differentiating between cracks and other defects which are generally limited only to the surface.

Based on the results of this preliminary investigation, the following simplified procedure can be recommended for the determination of approximate depths of shallow cracks in concrete pavements:

1. Use currently established procedures to acquire digital images of the pavement section to be evaluated;
2. Using a number of selected “benchmark” cracks in the above section, calibrate the model parameters \( p_a, p_v, \) and \( \alpha \) as described for different types of cracks (transverse and longitudinal) and joints;
3. Estimate the width \( a \) of any other crack of which the depth is desired based on the number of pixels in which an intensity contrast is seen in its image;
4. Knowing the crack width \( a \) and the pixel intensity of the uncracked area \( \Delta I_{(\text{max}ul)} \) plot the variation of the maximum theoretical pixel intensity reduction \( \Delta I_{\text{max}} \) with the crack depth \( d \) using Eq. (26) (for longitudinal cracks) or Eq. (27) (for transverse cracks); and
5. Select the depth that produces the maximum pixel intensity difference that closely matches the one obtained from the actual digital image. This is the approximate depth of the considered crack.
It is realized that the field implementation of the new technique is not completely assured at this preliminary phase of research. Also it must be emphasized that the new technique is not meant to replace any of the other image processing techniques that are currently in use, but rather it introduces a novel method of assessing the depths of cracks from images. As in the case of any other crack evaluation procedure, the technique must be improved by accounting for the random variations in pixel intensity, image noise, etc. When the current procedure is refined and programmed efficiently to make it more practical, it would provide an invaluable nondestructive means of evaluating crack depths in pavements and certainly supplement the existing crack evaluation techniques.

Conclusions

Variations in the BRDF at surface discontinuities were used to model digital images of longitudinal and transverse cracks and joints in concrete pavements. Ward’s isotropic model was chosen to express the BRDF because of its simplicity and the parameters that are physically meaningful. The theoretical formulation was simplified by assuming homogeneity and isotropy in concrete pavement surfaces and regular geometry in surface discontinuities. It was shown that the reduction in pixel intensities which produces a color contrast at the discontinuities can be modeled successfully in order to obtain information on the depth of cracks. For the calibration and experimental verification of the model, experimental cracks of varying severity and depths were formed in a concrete pavement. In the calibration process, the model parameters (diffusive and specular reflectances and the local slope variation) were determined by comparing the pixel intensity distributions of the modeled cracks and joints with their experimental counterparts. It was found that invariant model parameters could be used to model cracks and joints on a given concrete pavement. The calibrated model was able to produce additional images of cracks and joints that resembled their experimental counterparts to a reasonable degree in terms of both contrast in appearance and pixel intensity distributions within cracks. Another attractive feature of the new formulation is that the maximum pixel intensity reduction within a crack could be expressed in terms of the crack width and depth using an analytical relationship. It was also illustrated how this analytical relationship provides a convenient basis for predicting the crack depths based on crack widths and the pixel intensity contrast within the crack, thus, precluding the need for a time-consuming processing. In the present study only crack widths up to 10 mm were modeled. Obviously one can foresee no issue with cracks of higher widths. In fact, the writers speculate even a better accuracy in predicting the depth of relatively wide cracks. However, for crack widths smaller than 1 mm (hairline cracks), the identification depends on the resolution of the camera and the accuracy would be lower due to the lower contrast in intensities within an adequate width. In order to obtain more accurate results the assumption of homogeneity must be replaced with a means of modeling the random variation in reflection properties particularly inside the cracks. The new technique shows a definitive potential as an expedient tool for the evaluation of shallow crack depths useful in rehabilitation decision-making and differentiation of cracks from other surface irregularities in open-graded surface courses and joints in concrete pavements. Hence it would readily supplement the existing crack evaluation software as an additional tool.

Acknowledgments

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References


